## Grasp the Language Grasp the Mathematics


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Dr. Leslie Garrison along with members of Valle Imperial Mathematics presented this workshop at ICME-10. Below is the workshop description used for the initial proposal. On page 3 is the workshop agenda with added references to page numbers in this packet. Please contact us if you have questions or need further details.

For students who have yet to gain fluency in a language other than their primary language, learning to solve mathematical word problems can be very difficult. A primary goal of this workshop, designed for teachers who teach both mathematics and language, will be to give teachers of students who are not fluent in the language of instruction a proven strategy that helps these students improve in both mathematical problem solving and language acquisition. Workshop participants will first analyze and discuss model problems that have been used with students and the resulting samples of student work at a variety of grade levels. Strategies that enhance and encourage student discussion related to problem solving as well as correcting language errors will also be shared.

The heart of the workshop starts with creating a grammar lesson within a mathematical word problem by inserting intentional grammatical and spelling errors. By first correcting language errors, students make sense of the context of the problem in order to get at exactly what mathematical questions are involved. Participants will discuss their created problems in groups to insure that problems have rich mathematical potential as well as being able to encourage written and oral language acquisition skills. Questioning strategies that lead to unpacking the mathematics of the problem will also be discussed. Workshop participants will come away with a set of problems and strategies that can immediately be used in their classrooms.

Day 1: Monday, 5 July, 2004 - 16.30-17.30

## Objectives

Participants will:

* Become aware of problems second language learners encounter when faced with mathematics word problem solving.
* Learn a proven daily math and language strategy to address these problems.
* Practice with the strategy using a common problem.
- Welcome
- Background and issues (see p.4)
- Overview of the strategy (p.5)
- Practice the strategy (p. 6)
- Closure


## Day 2: Tuesday, 6 July, 2004 - 16.30-17.30

## Objectives

Participants will:

* Further their knowledge of the daily math and language strategy by analyzing examples of participant work.
* Create or modify mathematical word problems to use with the daily math and language strategy.
- Welcome back
- Analyze examples of participant work (p.7)
- Create problems (pp. 8-12)
- Closure


## Background and Issues

## Problem solving requires thinking.

Thinking is based on language.
Our students have limited academic language facility in the language of instruction (English) and their primary language (Spanish).

Who has the time in mathematics class to teach English as well?

Teaching mathematics and English have some things in common.
Students learn mathematics and English better when they work on problems that have action.

Students learn mathematics and English better when they work in groups.
Students learn mathematics and English better when the curriculum is narrow and deep.

Many teachers ask, "How can I help my students?" when it comes to solving word problems. Teachers have been frustrated in trying things like: teaching key words, identifying and underlining the question, rereading the problem, and modeling similar problems with limited or little success. Compound these issues with students who are struggling to learn English, having difficulty understanding vocabulary and grammar within the problem, and the task of solving word problems becomes exceedingly more difficult. The question remains, "How can I help my students?" especially those who are English Language Learners.

Dr. Greg Ponce, a researcher for Valle Imperial Mathematics, began working with a third grade teacher, Carolina Segura, to address this perplexing question. Ms. Segura had recently completed a 40 hour Institute in Cognitively Guided Instruction and was looking for ways to help her students who, were all English Language Learners having Spanish as their primary language, and having difficulty solving math word problems. Since Ms. Segura was already using Daily Oral Language (DOL) exercises each morning, Dr. Ponce and Ms. Segura decided to try combining DOL with mathematics problem solving.

The idea was to take mathematics word problems and create deliberate mistakes in spelling, grammar, punctuation, etc. for the students to first correct and discuss. With language hurdles having been cleared, students would then be better able to get at the mathematics in the problems. Thus, each morning when students entered the classroom they were greeted with two problems on the board similar to:

Mat have for stack of cards there are ate card in each stack how many cards are there in all
While Ms. Segura took attendance, lunch count, etc. students worked at making corrections. After five to ten minutes, Ms. Segura would facilitate an exchange of ideas concerning corrections. Typically one student at a time would be chosen to share a correction, and then the class decided whether she should show the correction on the board. If there was no agreement, then a discussion took place until the correction was agreed on and put on the board. The process was continued until all mistakes were corrected. Students corrected any existing errors in their initial work and then copied the corrected version of the problem into their math journals. It is important to note that students should not copy the incorrect version of the problems from the board, but should first attempt writing a corrected version for themselves. After all of the corrections are agreed on, only a corrected version goes into the math journal.

Next, students would work independently in trying to solve the math problem. After a few minutes they were encouraged to discuss solution strategies with a partner or group. Then selected individuals were asked to share their thinking and strategy with the class giving the entire class a chance to ask questions and Ms. Segura the opportunity to ask questions to deepen understanding.

Dr Ponce and Ms. Segura found that by asking students to first focus on language in order to help them understand the mathematics behind the language, students became much better at reading and understanding the problems, and thus became much better at being able to solve word problems.

An article by Ponce and Garrison entitled, "Overcoming the Wall of Silence and Despair that Surrounds Word Problems," will be published in Teaching Children Mathematics, by NCTM in December 04 or January 05 . The article contains details and ideas for implementing the strategy.

Joshua hav 34 plant. she gave some to Dennis. now Joshua has 14 plants lef
How many plant did Joshua giv to Dennis?
$5^{\text {th }}$
Mrs Martin used 3/8 of an bag of flower to baked cakes and $1 / 5$ of the rest of the bag to bake cokies what fraction of the hole bag did he had left

Mr. Silva bought 4 boxes of cookies. There were 12 chocolate cookies and 8 peanut cookies in each box. How many cookies were there altogether?

At the annual Yo-yo festival, all the yo-yo experts bring their best yo-yos. The festival also has a box of spare yo-yos. Last year, there were 34 yo-yos in the spare box. There were 27 experts and each one brought five yo-yos. How many yo-yos were there altogether?

## Kindergarten Problems:

Teresa has 14 dolls. Her friend Martha has 11 dolls. Who has more dolls?
Maria has 4 candies and Susana has 5 candies. How many candies do they have in all?
Mrs.Urias found 3 bags. Each bag had 5 candies in it. How many candies is that?
A cow has 4 legs. How many legs do 2 cows have?
Mary has 3 books to read. She lets Jenny have 1 of them to read. How many books does Mary have left to read?

5 rabbits were playing in a grassy area. 2 rabbits went off to find some food. How many rabbits are left in the grassy area?

5 bees are on a flower. 3 bees fly away. How many bees are left on the flower?
Gary started a tower with 3 blocks, and then he added 1 more block. How many blocks did he use in all?
There are 2 bananas in the basket. There are 3 more lying beside the basket. How many bananas are there altogether?

If you have 8 pennies and you find 1 more penny, how many pennies will you have in all?
There are 2 boys and 3 girls standing in front of the classroom. How many children are standing in front of the classroom?

Today 3 children want to play in the swings. The teacher said 2 more children can play in the swings. How many children in all can play in the swings?

If you have 4 unifix cubes and take away 4 unifix cubes. How many unifix cubes do you have left?
There are 3 children swinging on the swings. 1 child left to go get a drink of water. How many children are still on the swings?

## First Grade Problems

There were 6 birds.
Some birds went in the birdhouse.
4 birds are left. How many birds went in the birdhouse?
There were 10 bees.
Some went in the hive.
There are 6 bees left.
How many went in the hive?
Blanca has 7 white rabbits.
Then she got 3 brown rabbits.
How many rabbits are there?
5 bugs are on a leaf.

3 more join them.
How many bugs are there?
8 bugs are on a leaf.
3 bugs fly away.
Now how many bugs are there?
Jose had 7 pencils.
He gave 2 pencils away.
How many pencils does he have now?
Robert had 10 candies.
He ate some for a snack.
Now he has 6 left. How many did he eat for a snack?
Jose had 7 pencils.
He gave 2 pencils away.
How many pencils does he have now?
Kathy has 9 kittens.
She gave away some kittens.
Now she has 6 kittens.
How many kittens did she give away?
Manuel had some toy cars.
He got 3 more.
Now he has 8 cars.
How many cars did Manuel start with?
There are 5 dog bones in a bag.
Two bones are red.
The rest are green.
How many are green?

## Second Grade Problems

Jack rode his bike for 20 minutes on Monday. He rode it for 30 minutes on Tuesday. How many minutes did he ride his bike in all?

Billy is 16 years old. Paul is 6 years younger than Billy. How old is Paul?
Sam bought 20 m of rope. He used 7 m of it to make a swing. How much rope was left?
There are 6 chairs in each row. How many chairs are there in 3 rows?
Mrs. Gonzales bought some cans of sardines for $\$ 16$. Each can of sardines cost $\$ 2$. How many cans of sardines did she buy?

A supermarket is open from 11:00 a.m. to 9:00 p.m. every day. How many hours a day is the supermarket open?

## Third Grade Problems

429 concert tickets were sold on Sunday. 64 more concert tickets were sold on Sunday than on Saturday. How many tickets were sold on Saturday? 365

There are 8 buttons on each card. How many buttons are there on 5 cards? 40
There were 27 desks to clean. 3 boys shared the work equally. How many desks did each boy clean?
Mrs. Holmes bought 4 boxes of cookies. There were 12 chocolate cookies and 8 peanut cookies in each box. How many cookies were there altogether? 80 cookies

Sue ate $2 / 6$ of a pie. Frank ate _ of the pie. Who ate the bigger portion of the pie? Frank_ $=3 / 6$
Mr. Hartung took 2 h 35 min to repair a van and 1 h 55 min to repair a car. How long did he take to repair both vehicles? 4 h 30 min

## Fourth Grade Problems

The total cost of a scooter and 2 motorcycles is $\$ 9798$. The cost of each motor cycle is $\$ 3654$. Find the cost of the scooter. $\$ 2490$

3000 school books are arranged into 3 piles. The first pile has 10 more books than the second pile. The number of books in the second pile is twice the number of books in the third pile. How many books are there in the third pile? 598

Carl spent _ of his money on a storybook. If the storybook cost $\$ 6$, how much money did he have at first? \$24

Mrs. Page bought 5 potted plants. Each potted plant cost $\$ 2.35$. She gave the cashier $\$ 20$. How much change did she receive? $\$ 8.25$

Ali spent $\$ 3$ on string and $\$ 1.40$ on beads to make one pot hanger. How much will it cost him to make 4 pot hangers? $\$ 17.60$

## Fifth Grade Problems

Peter has twice as many stickers as David. How many stickers must Peter give David so that they each will have 120 stickers? 40 stickers

Mr. Li bought 155 oranges for $\$ 35$. He found that 15 of them were rotten. He sold all the remaining oranges at 7 for $\$ 2$. How much money did he make as profit? $\$ 5$

70 pupils were divided into 14 teams. In each team there were 2 girls. How many boys were there altogether? 42 boys
$3 / 7$ of the apples in a box are red apples. The rest are green apples. There are 24 green apples. How many apples are there all together? 42 apples

Mrs. Sanchez used $3 / 8$ of a bag of flour to bake cakes and $1 / 5$ of the remainder to bake biscuits. What fraction of the flour did she use altogether? $1 / 2 \mathrm{bag}$

## Sixth Grade Problems

Mrs. Item's class played a total of 38 kickball games. They won 11 more games than they lost. Three games ended as ties. How many games did Mrs. Item's class win? 23 games

Jon's bicycle has tires with a 26 inch diameter. How many revolutions with the tire need to make to travel 100 yards? 45, or a little more than 44 revolutions are needed

Keith is making a cover for a dish antenna. If the radius of the dish is 3 meters, what size will he have to make the cover? $28.26 \mathrm{~m}^{2}$

Marina is going to bake apple muffins. She has 20 apples and each muffin will use $2 / 3$ of an apple. How many muffins can she make? 30 muffins

Robert is selling brownies at the school picnic. There are 24 brownies in a pan. He has $2 / 3$ of a pan left. Mr. Thomas wants to buy _ of those remaining brownies. How many brownies will Mr. Thomas get? 8 brownies

## Seventh Grade Problems

Julian invested $\$ 5000$ in a special savings account that pays $8 \%$ annual interest compounded quarterly. How long must Julian keep this money in the account to earn at least $\$ 250$ in interest? 9 months

There are 30 students in the class. Eight of them are girls. What is the ratio of boys to students in the class? 22:30

If Bob completed $2 / 3$ of a cross country ski course and Jim completed $3 / 5$ of the course, which boy completed more of the course? Bob

The Student Council sold $75 \%$ of the tickets printed for the spring dance. If 120 tickets remain unsold, how many tickets were printed? 480 tickets

A freight train starts a trip from Tokyo at 12 noon. At 2:00 p.m. a passenger train starts over the same route at a speed 30 mph faster than the freight train. The passenger train overtakes the freight train at 6:00 p.m.. Find the average speed of each train. The freight averages 60 mph and the passenger train averages 90 mph .

## Algebra 1 Problems

Movie Theater It takes you 17 minutes to get to the movie theater. You spend $t$ minutes riding the bus at an average speed of 0.5 mile per minute. The rest of the time is spent walking at 0.06 mile per minute. If you spend ten minutes on the bus, how far is it to the movie theater? About 5.4 miles

Science Link The temperature within the Earth's crust increases about $30^{\circ} \mathrm{C}$ for each kilometer beneath the surface. If the temperature at the Earth's surface is $24^{\circ} \mathrm{C}$, at what depth would you expect the temperature to be $114^{\circ} \mathrm{C}$ ? 3 kilometers

Health Club Costs A health club has two payment plans. You can become a member by paying a $\$ 10$ new member fee and use the gym for $\$ 5$ a visit. Or, you can use the gym as non-member for $\$ 7$ a visit. Find the number of visits for which the plans would cost the same. 5 visits cost $\$ 35$ on either plan

Ladder The top of a ladder is 12 feet from the ground. The base of the ladder is 5 feet left of the wall. What is the slope of the ladder? Make a drawing to help you. 12/5

Tickets In one day a movie theater collected $\$ 4275$ from 675 people. The price of admission is $\$ 7$ for an adult and $\$ 5$ for a child. How many adults and how many children were admitted to the movie theater that day? 450 adults and 225 children

T-Shirt Cannon At a basketball game, T-shirts are rolled-up into a ball and shot from a "T-shirt cannon" into the crowd. The T-shirts are released from a height of 6 feet with an initial upward velocity of 44 feet per second. If you catch a T-shirt at your seat 30 feet above the court, how long was it I the air before you caught it? Is your answer reasonable?

